Wind Power

Calculation Exercise (approximately 60 min)

Task Description

When comparing the cards that feature electric and diesel cars, electric cars appear to be more climate friendly. However, this depends on how the electricity used to power electric cars is generated, which varies between countries. Electricity can be generated from for example hydropower, nuclear power, wind power, or from the burning of fossil fuels such as coal or natural (fossil) gas.

Wind now meets ca 17% of Europe's electricity demand and much more in some countries, according to Wind Europe. Wind power is being increasingly developed, and is likely going to play an even more important role in the electricity mix of the future.

This task involves examining how wind power functions and what would be required to replace a nuclear power plant with wind power.

Betz's Law is a fundamental principle in wind power technology. It states that the theoretically maximum power that can be extracted in a wind turbine is 16/27 (or approximately 59%) of the energy of the air passing through the wind turbine:

$$P = \frac{16}{27} \cdot \frac{\rho \cdot \pi \cdot r^2 \cdot v^3}{2}$$

P is the maximum power [in W] that can be extracted by the wind turbine, ρ [rho; kg/m^3] is the air density, r [m] is the radius of the wind rotor (that is, the length of the rotor blades), and v [m/s] is the wind speed at the rotor's height.

The wind speed, v, varies at different heights above the ground. The value reported in weather forecasts often refers to wind speed at a height of 10 meters, but wind turbines are much taller than that. An approximate value for the wind at a height of *h* meters above the ground is given by the relationship:

$$v = v(h) = v(10) \cdot \left(\frac{h}{10}\right)^{0.16}$$

where v(10) is the wind speed at 10 meters above ground level.

a) Use unit analysis to control that the left and right sides of Betz's law have the same units.

b) Calculate the wind speed at a height of 50 meters using the formula above. Assume that v(10) = 15, meaning that the wind speed at 10 meters height is 15 m/s.

c) Calculate the maximum theoretical power output of a wind turbine. Find the necessary values for the calculation or make reasonable assumptions. You can choose whether to calculate the power for a small, medium-sized, or large wind turbine. Make sure to express the result in the correct unit!

d) Estimate the number of wind turbines needed to replace a nuclear power plant. Find the values required for the calculation, or make reasonable assumptions.

e) A typical medium-sized wind turbine generates approximately 5.1 GWh per year. How does this compare to the value you calculated in c)? Reflect on the possible reasons for any differences and calculate how many wind turbines would be needed to replace a nuclear power plant if all wind turbines produced 5.1 GWh per year.

f) The use of nuclear power has been a long-standing controversial issue. Reflect on your own and attempt to answer the following questions in as much detail as possible: What are the advantages and disadvantages of nuclear power and wind power? Why is it challenging to entirely replace nuclear power with wind power?

Suggested Solutions

a) Unit analysis of the variables in Betz's law:

- P represents power, which is measured in Watt (W).
- ρ represents air density, which is measured in kilograms per cubic meter (kg/m^3).
- r represents radius, which is measured in meters (m). The radius is the distance from the center of the wind turbine's rotor to the edge of the rotor blade.
- v represents wind speed, which is measured in meters per second (m/s).

The left-hand side of Betz's law is in units of W (Watt). The right-hand side of Betz's law has units of J/s (Joules per second), which is shown as follows:

$$\frac{kg}{m^3}\cdot m^2\cdot\left(\frac{m}{s}\right)^3 = \frac{kg\cdot m^2\cdot m^3}{m^3\cdot s^3} = \frac{kg\cdot m^2}{s^3} = \frac{J}{s}$$

This follows from the fact that Joule is a *derived SI unit*, which can also be expressed as: J = $m^2\cdot kg\cdot s^{-2}$. The left and right sides, therefore, have the same units because W = J/s.

b)

$$v(50) = 15\cdot\left(\frac{50}{10}\right)^{0.16} \approx 19.4\ m/s$$

The calculation shows that the wind speed at a height of 50 meters is a little over 19 m/s when the wind is blowing at 15 m/s at a height of 10 meters.

c) Medium-sized wind turbines for commercial use typically have rotor blades with radius that range from 20 to 60 meters. The most powerful wind turbines used in larger wind farms can have rotor blades with a radius of over 100 meters. Let's assume that this wind turbine has a height (h) of 150 meters, a radius (r) of 50 meters, and that v(10)=15 m/s.

First, the wind speeds at a height of 150 meters is calculated:

$$v(150) = 15 \cdot \left(\frac{150}{10}\right)^{0.16} \approx 23.1\ m/s$$

Next, the theoretically maximum power is calculated using Betz's law. The density of air is approximately 1.2 kg/m^3.

$$P = \frac{16}{27} \cdot \frac{1.2 \cdot \pi \cdot 50^2 \cdot 23.1^3}{2} \approx 34.4\ MW$$

d) The energy output from a nuclear power plant depends on several factors, including the type of reactor, its capacity, operating time, and efficiency. Here, we assume that a nuclear power plant produces 7 TWh per year (equivalent to 7000 GWh per year), which is a typical value for a larger nuclear power plant.

The theoretically maximum electricity production (E) of a wind turbine over one year is calculated by multiplying the theoretically maximum power (P) by the time (t) in hours per year:

$$E = P \cdot t = 34.4\ MW \times 24\ hours/day \times 365\ days/year \approx 301\ GWh/year$$

Next, the number of wind turbines required to replace a nuclear power plant is calculated:

$$Number\ of\ wind\ turbines = \frac{7\ 000\ GWh/year}{301\ GWh/year} \approx 23$$

The calculation shows that 23 wind turbines are needed to replace a nuclear power plant generating 7 TWh per year.

e) The calculated value (301 GWh/year) is significantly larger than the average value for a real wind turbine (5.1 GWh/year). This is partly because the wind doesn't always blow at the speed necessary to reach peak efficiency, and partly because we have neglected all friction losses.

The number of wind turbines needed to replace a nuclear power plant if all wind turbines produced 5.1 GWh per year is calculated as follows:

$$Number\ of\ wind\ turbines = \frac{7\ 000\ GWh/year}{5.1\ GWh/year} \approx 1372$$

f) Nuclear power and wind power each have their pros and cons. While wind power offers clean energy, its output varies with wind conditions, posing challenges for consistent supply. Immediate use or storage is essential, matching demand precisely to avoid conversion losses. Nuclear power is environmentally friendly but costly to construct and faces security and waste disposal concerns.

Replacing nuclear power with wind power entirely is challenging. Wind's variability necessitates complementary energy sources, like hydropower, for stability. Wind's inherent unpredictability complicates grid management. Nuclear power provides a stable, continuous energy supply. Transitioning requires infrastructure upgrades and investment in energy storage technology.

Expected Learning Outcomes

Students gain understanding of wind power technology, apply unit analysis to analyze physical relationships such as Betz's law, convert units and calculate wind speed variations with height. These outcomes are intended to deepen their comprehension of wind power and its practical physics applications. The idea behind the final question is to provide an open task where students can make arguments based on their calculations and other relevant facts.

To make the task easier, as a teacher, you can choose to provide students with all the values they need for the calculations.